

Two resistances were inserted in the circuit for two different sets of readings, in the first case 1.1 ohm of total resistance, in the other 2.72.

Table VI refers to these experiments, the curve marked A in the Diagram V being that due to the first, B being that due to the second. It will be noticed that practically the curve becomes a straight line, or, in other words, the electromotive force increases directly as the number of revolutions of the armature, a result which might have been expected theoretically. It will also be seen that the current for any given number of revolutions varies inversely as the resistance in circuit, or, in other words, that the electromotive force for a given number of revolutions is constant.

In any results, therefore, which are given descriptive of the light produced by any machine, the following should be noted :—

Number of revolutions of armature ;  
Resistance in circuit ;  
Horse-power expended ;  
Colour of light measured ;  
Electromotive forces.

together with the size of the carbons employed, and other obvious details.

II. "Experimental Researches on the Temperature of the Head." By J. S. LOMBARD, M.D., formerly Assistant-Professor of Physiology in Harvard University, U.S. Communicated by H. CHARLTON BASTIAN, M.D., F.R.S., Professor of Pathological Anatomy in University College, London.

(Abstract.)

The present communication forms an abstract of the first portion of a series of investigations having the following primary objects in view :—

1st. To find out, as far as possible, the normal relative temperatures of different portions of the surface of the head, when the brain is comparatively inactive.

2nd. To study the effect of different mental states upon the different portions of the surface of the head previously examined in the condition of comparative cerebral inactivity.

The *ultimate* objects were two-fold ; namely :—

1st. To furnish, if possible, some reliable data as a starting point,

for examining the temperature of the surface of the head, with a view to assisting in the diagnosis and localization of cerebral disease.

2nd. To see if, from an examination of the relative temperatures of different portions of the surface of the head during increased mental activity, any information could be obtained as to the comparative importance of the parts played by different portions of the brain in the evolution of thought and the different emotions.

To commence with the first of these ultimate objects of the investigations, it is easy to see that everything depends upon an accurate knowledge of the normal *relative temperatures* of the different portions of the surface of the head, and the variations of such temperatures within healthy limits. Without this knowledge it is impossible to come to any satisfactory conclusion as to the existence of localized cerebral disorder by examining the temperature of the surface of the head.

So far as the writer is aware, Dr. Wm. A. Hammond,\* of New York, was the first to indicate that a difference of temperature exists between the two sides of the head in health. In 1875, Dr. Hammond, making use of a thermo-electric apparatus, devised by the writer the year previous, came to the conclusion, from observations made on a large number of individuals, that the left side of the head has a higher temperature than the right side. Unfortunately the notes furnished by Dr. Hammond, in the writer's possession, do not state the *exact locality* examined, a matter of great importance as will be seen further on.

During the past year M. Broca† has brought forward experiments on the same subject. M. Broca, using thermometers, has also come to the conclusion that the left side of the head has the higher temperature. The sources of error in M. Broca's method will be presently noticed.

Turning now to the *second* of the ultimate objects of the present investigations, namely, the connexion between the relative share of mental work done in a given part of the brain, and the relative temperature of the surface over such part, it is evident that here also a thorough acquaintance with the relative temperatures of the different portions of the surface of the head in the quiescent mental condition is requisite.

As regards what has been already done on the subject of the relation between heat and mental work, it may be summed up in a few words.

In 1866 the writer commenced a series of experiments with thermo-electric apparatus on the effect of increased mental activity on the temperature of the head. These experiments showed that the exercise

\* Dr. Hammond's paper was read before the New York Neurological Society, October 4, 1875.

† "Revue Scientifique," September 15, 1877.

of the higher intellectual faculties, as well as different emotions, caused a perceptible rise of temperature in the head. Merely arousing the attention could produce the same result. These results were published in June of 1867.\* Toward the close of the latter year, Professor Moritz Schiff, who had been working independently of any knowledge of what the writer had been doing, communicated to the Museum of Natural History of Florence results of a similar nature. In 1870 Professor Schiff † published an account of a series of investigations made directly upon the brain of animals, which decisively proved that mental work is accompanied by elevation of temperature in the brain. Lastly, M. Broca has likewise arrived at the same conclusion by experiments made, like the writer's, on the human subject, M. Broca, however, using thermometers instead of thermo-electric apparatus.

The present investigations were commenced in January, 1877, and have been continued almost without interruption to the present time. The first step to be taken having been decided upon, namely, a thorough examination of the normal relative temperatures of the different portions of the surface of head, the next point to be decided was the manner in which the examination could be best carried on. Preliminary observations had satisfied the writer that experiments made upon the heads of individuals taken at random could only lead to confusing and contradictory results. Accordingly, the investigations were limited to a few selected heads, which could be measured and compared, and the different circumstances, both internal and external, affecting which, could be pretty well known. Six subjects, three males and three females, were selected. The next question was the measurement of the head, and its division into regions, and the subdivision of these regions. The following method was decided on: the head was divided into three regions, designated respectively, *anterior*, *middle*, and *posterior*.

#### *Anterior Region.*

The anterior region is bounded laterally by a line drawn upward, on each side of the said, from the angle formed by the frontal and zygomatic processes of the malar bone in a direction parallel to the plane of the forehead, taken over the frontal eminences and superciliary ridges.

The superior boundary is formed by the continuation in the same plane, and junction on the top of the head, of the lateral boundaries.

The inferior boundary is formed by a line passing horizontally across the front of the head on a level with the summits of the supra-orbital arches between the external angular processes of the two sides, and

\* "New York Medical Journal," June, 1867, and "Archives de Physiologie," September—October, 1868.

† "Archives de Physiologie," t. iii, p. 6, 1870.

thence continued by the outer borders of the malar bones to the points of origin of the lateral boundaries.

The part of the head embraced by the anterior region is, therefore, that which would be cut off anteriorly by a transverse and vertical section, made parallel to the plane specified, between the angles formed by the frontal and zygomatic processes of the malar bones of the two sides.

#### *Middle Region.*

The middle region is bounded anteriorly by the lateral and superior boundaries of the anterior region.

The posterior boundary is formed by a line passing over the top of the head parallel to the anterior boundary, uniting the extremities of the mastoid processes of the two sides.

The inferior boundaries commencing at the terminations of the inferior boundary of the anterior region, follow the upper borders of the zygomatic processes of the temporal bones, pass behind the ears, and follow the anterior borders of the mastoid processes to their extremities.

#### *Posterior Region.*

The posterior region has for its superior and lateral boundaries the posterior boundary of the middle region.

The inferior boundary is formed by the posterior borders of the mastoid processes and by the superior curved line of the occipital bone.

The longitudinal median line of the head divides each of the three regions into right and left symmetrical halves.

In the present communication we have only to do with the anterior region.

The anterior region is thus subdivided:—

Commencing at the inferior boundary, each lateral half is divided into six parts by five equidistant horizontal lines drawn from the median line to the lateral limit. The tracts thus marked off are designated *tiers*, and are numbered from 1 to 6, commencing at the inferior boundary. Further in each lateral half four equidistant vertical lines are drawn upward parallel to the median line, from the lower limit to the superior boundary, thus dividing each *tier* into smaller spaces. In the first four tiers these spaces are five in number; but in the fifth and sixth tiers, the convergence, over the top of the head, of the lateral limits diminishes the length of the tier, and reduces the number of spaces to four in the fifth tier and to three in the sixth tier. These spaces are designated *districts*, and are numbered from 1 to 5 outward from the median line. The tiers are marked off by means of a string coated with coloured chalk.

As the method of measuring off the tiers and districts by equidistant

lines does not in every head bring a given space into exactly the same anatomical position, each case is referred to one head as a standard. The following are the measurements and anatomical positions of the tiers in the standard head.

The height of the region measured on the median line is 125 mm. (4.92 inches), therefore, each of the six tiers measures 20.83 mm. (0.82 inch) vertically. The upper boundary of the first tier touches the summits of the superciliary ridges. The upper boundary of the second tier passes through the centre of the frontal eminences. The upper boundary of the third tier touches the upper border of the frontal eminences. The upper boundaries of the fourth and fifth tiers have no anatomical landmarks, and their positions can be designated only by their respective distances from the superior limit of the region; this latter touches the coronal suture on the median line, hence the upper boundaries of the fourth and fifth tiers are, respectively, 41.66 mm. (1.64 inch), and 20.83 (0.82 inch) distant from the coronal suture on the median line.

The following are the measurements of breadth :—

Total breadth measured on the horizontal portion of the inferior boundary of the region 186 mm. (7.32 inches).

Total breadth measured over middle of frontal eminences 186 mm. (7.32 inches).

Total breadth measured just above frontal eminences 176 mm. (6.928 inches).

Total breadth measured at a distance of 20.83 mm. (0.82 inch) from the coronal suture on the median line 120 mm. (4.72 inches).

The breadth of each lateral half measured on the inferior boundary line being 93 mm. (3.66 inches), each of the five districts will measure on this line 18.6 mm (0.732 inches). We have then 27 spaces a side to examine, the maximum size of the space being 21 mm. (0.8 inch) by 19 mm. (0.7 inch).

With reference to the instruments employed, both thermometers and thermo-electric apparatus have been used. The writer does not, however, think thermometers reliable in investigations of this kind; for the reason that they cannot be pushed down firmly enough upon the surface to empty the superficial vessels. The piles used by the writer are set in paraffine in an ebonite casing so as to be at some distance from the edges of the casing, and the whole bottom,—faces of piles, paraffine, and ebonite edges,—rendered perfectly flush. Pressure with a pile so constructed empties the superficial vessels, leaving the part pale and bloodless on the removal of the pile. It is easy with such piles to empty the temporal artery and to test the temperature of parts lying beneath it without the least risk of the temperature of the blood in the artery itself interfering. The danger of error in experiments such as those of M. Broca, is that the tempera-

ture of the blood of the superficial vessels comes to affect the thermometer, as well as the temperature of the deeper seated parts.

Sir W. Thomson's galvanometer, and one devised by the writer himself, already described several years ago, were employed.\* The rheostat and keys have also been described in 1868.†

We will now proceed to examine the different spaces of the anterior region.

*1st. Comparison of symmetrically situated spaces of the two sides of the head, 100 observations on each pair of spaces.*

The first fact of importance demonstrated by this examination is, that in *no one of the spaces into which the anterior region is subdivided is the temperature uniformly higher on one side than on the other: on the contrary, it may be higher on the right side or on the left side in turn.* We have, therefore, to consider only on which side of the head in the majority of cases the higher temperature is found in a given pair of spaces. The following is the general distribution of temperature:—

In favour of			
<i>Left side.</i>		<i>Right side.</i>	
<i>1st Tier.</i>		<i>1st Tier.</i>	
Districts—1st, 2nd, 3rd.		Districts—4th, 5th.	
<hr/>		<hr/>	
<i>2nd Tier.</i>		<i>2nd Tier.</i>	
„ 1st, 2nd, 3rd.		„ 4th, 5th.	
<hr/>		<hr/>	
<i>3rd Tier.</i>		<i>3rd Tier.</i>	
„ 1st, 2nd, 3rd.		„ 4th, 5th.	
<hr/>		<hr/>	
		<i>4th Tier.</i>	
		„ 1st, 2nd, 3rd, 4th, 5th.	
		<hr/>	
		<i>5th Tier.</i>	
		„ 1st, 2nd, 3rd, 4th.	
		<hr/>	
		<i>6th Tier.</i>	
		„ 1st, 2nd, 3rd.	
		<hr/>	

Thus in the 27 spaces a side compared, the average relative temperature is higher on the right side than on the left in 18 spaces, or two-thirds of the whole number.

Next, taking the total number of observations, 2700, and deducting

\* "British Medical Journal," January 23, 1875.

† "Archives de Physiologie," July—August, 1868.

220 cases of equality of temperature, to be presently considered, the following is the apportionment of the remaining 2480 results :—

In favour of right side .....	1343
„ „ left „ .....	1137

Hence the percentages of times of occurrence of relative superiority of temperature for the right and left sides respectively are 54·153 and 45·847. But in the nine spaces in which the left side has the majority of cases of higher temperature, this majority is greater than the majority found in the eighteen spaces in which the right side has the higher average : thus, in the nine spaces specified, the left side shows a mean percentage of 75·069 cases of superiority of temperature ; while in the eighteen spaces in which the right side predominates the mean percentage of cases of superiority of temperature is 68·117 ; that is to say, those spaces which, on an average, are higher in temperature on the left side are more exclusively so than those which, on an average, are higher in temperature on the right side.

Next, as regards equality of temperature of the two sides, the following are the spaces in which equality of temperature is most frequently found :—

	1st Tier.
Districts—1st, 2nd, 3rd.	
	-----
	2nd Tier.
„ 3rd, 4th.	
	-----
	3rd Tier.
„ 1st, 2nd, 3rd, 4th.	
	-----
	4th Tier.
„ 1st, 2nd, 3rd, 4th.	
	-----
	5th Tier.
„ 1st, 2nd, 3rd, 4th.	

Thus equality of temperature of the two sides is found in 16 spaces, or in 59·259 per cent. of the whole number of spaces. Of the total number of observations, 2,700, 220, or 8·148 per cent. show equality of temperature. Taking the total number of observations we have the following percentages of times of occurrence of superiority of temperature on the right side, on the left side, and of equality of temperature :

Right side .. ..	50·112 per cent.
Left „ .. ..	41·740 „ „
Equality .. ..	8·148 „ „

The five highest percentages in favour of the right side are distributed as follows :

5th District of 2nd Tier	..	— 76 per cent.
5th    "    "    3rd    "	..	— 70   "   "
2nd    "    "    5th    "	..	— 70   "   "
4th    "    "    5th    "	..	— 69   "   "
2nd    "    "    6th    "	..	— 71   "   "

The five highest percentages in favour of the left side are distributed as follows :

1st District of 1st Tier	..	— 74 per cent.
2nd    "    "    1st    "	..	— 77   "   "
1st    "    "    2nd    "	..	— 74   "   "
2nd    "    "    2nd    "	..	— 76   "   "
2rd    "    "    3rd    "	..	— 75   "   "

The five highest percentages of equality of temperature are distributed as follows :

4th District of 2nd Tier	..	— 22 per cent.
1st    "    "    3rd    "	..	— 22   "   "
2nd    "    "    3rd    "	..	— 24   "   "
2nd    "    "    4th    "	..	— 24   "   "
3rd    "    "    4th    "	..	— 30   "   "

The following shows the distribution of temperature in two cases of common occurrence, the one with increase in extent of the tract of superior temperature on the right side, and the other with increase in extent of the tract of superior temperature on the left side. These deviations from the general order of things often persist for some hours.

*Increase of tract of Right Superiority of Temperature.*

In favour of

Left side.	Right side.
<i>Districts.</i>	<i>Districts.</i>
1st Tier—1st, 2nd,	3rd, 4th, 5th.
2nd Tier—1st, 2nd,	3rd, 4th, 5th.
3rd Tier—1st, 2nd, and part of 3rd,	part of 3rd ; 4th, 5th.
4th Tier—	1st, 2nd, 3rd, 4th, 5th.
5th Tier—	1st, 2nd, 3rd, 4th, 5th.
6th Tier—	1st, 2nd, 3rd, 4th, 5th.

*Increase of tract of Left Superiority of Temperature.*

In favour of	
Left side.	Right side.
<i>Districts.</i>	<i>Districts.</i>
1st Tier—1st, 2nd, 3rd, 4th,	5th.
2nd Tier—1st, 2nd, 3rd, 4th,	5th.
3rd Tier—1st, 2nd, 3rd, 4th,	5th.
4th Tier—1st, 2nd, 3rd, 4th,	5th.
5th Tier—parts of 1st, 2nd, 3rd,	parts of 1st, 2nd, 3rd; 4th, 5th.
6th Tier—	1st, 2nd, 3rd, 4th, 5th.

Before leaving this part of the subject it may be well to state that the situation of M. Broca's "frontal" thermometer seems to have been in the neighbourhood of the 5th district, 1st tier, so far as the writer can judge from the abstract of M. Broca's paper, which has come under his (the writer's) notice. Now, for this space, the writer's results are, 68 in favour of the right side, and 32 in favour of the left side; but immediately adjoining is the 1st district 1st tier of the writer's *middle region*, and, in the space specified, the figures are 30 right, 60 left, and 10 neutral. M. Broca's thermometer may have been in this space. The examination of the *middle region* does not enter into this communication, and the above remarks are made principally to show within what narrow limits the balance of superiority of temperature may shift from one side to another.

*Quantitative comparisons of the two sides.*

The following is a summary of the mean results of 100 examinations of each pair of symmetrically situated spaces.

The mean difference of temperature is pretty nearly the same for both sides of the head; thus, the mean difference of temperature for the eighteen spaces which are, on an average, of higher temperature on the right side than on the left, is  $0.255^{\circ}\text{C.}$  ( $0.459^{\circ}\text{F.}$ ); while the mean difference of temperature for the nine spaces which are of higher temperature on the left side than on the right, is  $0.2411^{\circ}\text{C.}$  ( $0.433^{\circ}\text{F.}$ ). The greatest difference noted is in the 3rd district, 3rd tier, left side, namely,  $0.461^{\circ}\text{C.}$  ( $0.829^{\circ}\text{F.}$ ); the smallest differences noted are in the 1st district, 4th and 5th tiers, and in the 2nd district, 4th tier, all right side, the differences being each  $0.076^{\circ}\text{C.}$  ( $0.136^{\circ}\text{F.}$ ). The extreme range of difference of temperature is therefore  $0.385^{\circ}\text{C.}$  ( $0.693^{\circ}\text{F.}$ ). The mean difference of temperature of all the observations taken together, irrespective of sides, is  $0.247^{\circ}\text{C.}$  ( $0.444^{\circ}\text{F.}$ ).

*2nd.—Comparison of spaces situated on one and the same side of the Head.*

(a.) Comparison of spaces situated in the same district of two adjoining tiers—50 observations on each pair of spaces.

The following is a summary of the principal results;

1st. The whole of the 2nd tier is, in the majority of cases, of higher temperature than the 1st tier, on both sides of the head; with the exception, that on both sides, the 1st district is generally of higher temperature in the 1st tier than in the 2nd tier.

2nd. The whole of the 2nd tier is, in the majority of cases, of higher temperature than the 3rd tier, on both sides of the head; but this majority is much less than that possessed by the 2nd tier over the 1st tier.

3rd. The whole of the third tier is, in the majority of cases, of higher temperature than the 4th tier, on both sides of the head.

4th. The whole of the 4th tier is, in the majority of cases, of higher temperature than the 5th tier, on both sides of the head.

5th. The whole of the 6th tier is, in the majority of cases, of higher temperature than the 5th tier, on both sides of the head.

The mean quantitative results are as follows:

*Left side.*

*Right side.*

1st district, 1st tier, superior in temperature to

1st district, 2nd tier, by  $0.03^{\circ}\text{C.}$  ( $0.054^{\circ}\text{F.}$ )  $\dots 0.04^{\circ}\text{C.}$  ( $0.072^{\circ}\text{F.}$ ).

Remainder of 2nd tier, superior to 1st tier, by

$0.155^{\circ}\text{C.}$  ( $0.279^{\circ}\text{F.}$ )  $\dots\dots\dots 0.165^{\circ}\text{C.}$  ( $0.297^{\circ}\text{F.}$ )

2nd tier, superior to 3rd tier, by  $0.08^{\circ}\text{C.}$  ( $0.144^{\circ}\text{F.}$ )  $\dots 0.068^{\circ}\text{C.}$  ( $0.122^{\circ}\text{F.}$ )

3rd tier, superior to 4th tier, by  $0.296^{\circ}\text{C.}$  ( $0.532^{\circ}\text{F.}$ )  $\dots 0.324^{\circ}\text{C.}$  ( $0.583^{\circ}\text{F.}$ )

4th tier, superior to 5th tier, by  $0.312^{\circ}\text{C.}$  ( $0.561^{\circ}\text{F.}$ )  $\dots 0.302^{\circ}\text{C.}$  ( $0.543^{\circ}\text{F.}$ )

6th tier, superior to 5th tier, by  $0.26^{\circ}\text{C.}$  ( $0.468^{\circ}\text{F.}$ )  $\dots 0.273^{\circ}\text{C.}$  ( $0.491^{\circ}\text{F.}$ )

It will be seen from the above figures, that the difference between spaces of adjoining tiers on one and the same side of the head is nearly as great as the difference between symmetrically situated spaces of the two sides. The order in which the tiers come, as regards their temperatures, is as follows:

2nd tier—3rd tier—1st tier—4th tier—6th tier—5th tier; this order holding good for both sides of the head.

(b.) Comparison of spaces situated in two adjoining districts of the same tier, 50 observations on each pair of spaces.

The following is a summary of the principal results:—

1st. The 1st district is of higher temperature than the 2nd district, in the majority of cases, in the 1st and 5th tiers, on both sides of the head.

2nd. The 2nd district is of higher temperature than the 1st, in the majority of cases, in the 2nd, 3rd, 4th, and 6th tiers, on both sides of the head.

3rd. The 3rd district is of higher temperature than the 2nd district, in the majority of cases, in every tier, on both sides of the head.

4th. The 4th district is of higher temperature than the 3rd district, in the majority of cases, in every tier, on both sides of the head.

5th. The 4th district is of higher temperature than the 5th district, in the majority of cases, in the 1st tier, left side; in the 2nd and 3rd tiers, right side; and in the 4th tier, on both sides of the head.

6th. The 5th district is of higher temperature than the 4th district, in the majority of cases, in the 1st tier, right side; and in the 2nd and 3rd tiers, left side.

The mean quantitative results are as follows :—

<i>Left side.</i>	<i>Right side.</i>
1st district superior in temperature to 2nd district by $0.042^{\circ}$ C. ( $0.075^{\circ}$ F.)	} $-0.037^{\circ}$ C. ( $0.066^{\circ}$ F.)
3rd district superior to 2nd district by $0.053^{\circ}$ C. ( $0.095^{\circ}$ F.)	
4th district superior to 3rd district by $0.114^{\circ}$ C. ( $0.205^{\circ}$ F.)	} $-0.032^{\circ}$ C. ( $0.057^{\circ}$ F.)
4th district superior to 5th district by $0.02^{\circ}$ C. ( $0.036^{\circ}$ F.)	

From the above it can be seen that the mean difference between districts is much less than that existing between tiers. The order in which the districts come, as regards their temperatures, is as follows :—

4th district, 5th district, 3rd district, 1st district, 2nd district; this order holding good for both sides of the head.

Lastly, if we take the three groups of examinations, namely, those on the comparative temperature of the two sides of the head: those on the comparative temperature of adjoining tiers on one and the same side; and those on the comparative temperature of adjoining districts on one and the same side, we have the following values :—

*Comparison of the two sides of the Head.*

Average percentage of times of occurrence of superiority of temperature of either side of the head over the other.....	Average difference of temperature
61.074	— $0.247^{\circ}$ C. ( $0.444^{\circ}$ F.)

*Comparison of adjoining tiers of one and the same side.*

Average percentage of times of occurrence of superiority of temperature of one tier over another on both sides taken together .....	Average difference of temperature
65.506	— $0.21^{\circ}$ C. ( $0.378^{\circ}$ F.)

*Comparison of adjoining districts of one and the same side.*

Average percentage of times of occurrence of superiority of temperature of one district over another on both sides taken together .....	Average difference of temperature 65·219 — 0·055° C. (0·099° F.)
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According to the above figures, superiority of temperature of a given side of the head over the other, is a little less frequent of occurrence than superiority of temperature of a given tier or district of one and the same side over another tier or district. The two sides of the head are rarely equal in temperature, but the balance of superiority shifts so frequently from one side to the other, that the percentage of superiority for either side, in a given number of observations, is comparatively small.

III. "Addition to Memoir on the Transformation of Elliptic Functions." By A. CAYLEY, F.R.S., Sadlerian Professor of Pure Mathematics in the University of Cambridge. Received February 6, 1878.

(Abstract.)

I have recently succeeded in completing a theory considered in my "Memoir on the Transformation of Elliptic Functions," Phil. Trans., t. 164 (1874), pp. 397–456, that of the septic transformation,  $n=7$ . We have here

$$\frac{1-y}{1+y} = \frac{1-x}{1+x} \left( \frac{a-\beta x + \gamma x^2 - \delta x^3}{a+\beta x^2 + \gamma x^2 + \delta x^3} \right)^2,$$

a solution of

$$\frac{Mdy}{\sqrt{1-y^2} \cdot 1-v^8 y^2} = \frac{dx}{\sqrt{1-x^2} \cdot 1-u^8 x^2}$$

where  $\frac{1}{M} = 1 + \frac{2\beta}{a}$ ; and the ratios  $a:\beta:\gamma:\delta$ , and the  $uv$ -modular equation are determined by the equations

$$\begin{aligned} u^{14} a^2 &= v^2 \delta^2, \\ u^6 (2a\gamma + 2a\beta + \beta^2) &= v^2 (\gamma^2 + 2\gamma\delta + 2\beta\delta), \\ \gamma^2 + 2\beta\gamma + 2a\delta + 2\beta\delta &= v^2 u^2 (2a\gamma + 2\beta\gamma + 2a\delta + \beta^2), \\ \delta^2 + 2\gamma\delta &= v^2 u^{10} (a^2 + 2a\beta); \end{aligned}$$

or what is the same thing, writing  $a=1$ , the first equation may be replaced by  $\delta = \frac{u^7}{v}$ , and then,  $a, \delta$  having these values, the last three